## A STUDY OF LATERALLY LOADED PILE GROUPS USING THE QUASI-3D FINITE ELEMENT METHOD

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#### ABSTRACT

Full-scale lateral load tests on a group of bored and a group of driven precast (PC)

piles were carried out in Chaiyi Taiwan and the results were reported by Huang et al. (2001). Lateral response of the bored and PC pile groups were analyzed using the quasi-3D finite element approach (Wu, 1994; Wu and Finn, 1997a, 1997b) in a nonlinear analysis that uses strain-dependent soil moduli and damping, yielding at failure and no-tension cut-off. The response of the pile groups is computed directly, without having to use the p-y curves and the p-multipliers, but using only the fundamental parameters of the pile-soil system such as the geometry and stiffness of the pile, and stiffness and strength of the surrounding soils.

#### RÉSUMÉ

La réponse latérale des groupes de pieux préfabriqués et celle des pieux forés (Huang et al. 2001) ont été analysées en utilisant une approche d'éléments finis quasi tridimensionnelle (Wu 1994; Wu et Finn 1997a, 1997b) dans une analyse non linéaire. La réponse des groupes de pieux est directement calculée sans l'aide de graphiques de charge latérale vs la déflection, ni de facteur de réduction sur la charge latérale. Seuls les paramètres fondamentaux des systèmes pieux-sol sont utilisés, tels que la géométrie et la rigidité du pieu ainsi que la rigidité et la résistance du sol environnant.

#### 1. INTRODUCTION

Full-scale lateral load tests on a group of bored and a group of driven precast piles were carried out in Chaivi Taiwan and the results were reported by Huang et al. (2001). Numerical analyses of the laterally loaded single piles were conducted by Huang et al. (2001) using the p-y curves derived from in-situ DMT tests. Response of the pile groups were analyzed using the concept of pmultipliers which is essentially a factor to account for pilesoil-pile interaction or group effect. P-multipliers for both the bored and driven pile groups were back-calculated from the Chaiyi lateral group load tests and reported in Huang et al. (2001). Mostafa and El Naggar (2002) computed dynamic p-multipliers of the two pile groups using a combination of static p-y curves and the plane strain assumptions (Novak, 1974) to represent the soil reaction within the framework of a Winkler model, and compared their results with those by Huang et al. (2001).

A quasi-3D finite element method was developed by Wu (1994) and Wu and Finn (1997a, 1997b) to analyze the response of single pile and pile groups subject to dynamic lateral loads. The method is further enhanced by Wu (2006) to account for pile diameter effect. Energy transmitting boundary conditions are implemented for analysis of high-frequency vibration. Lateral response of the bored and drilled pile groups in Huang et al. (2001) are analyzed with the quasi-3D finite element approach in a nonlinear analysis that uses strain-dependent soil modulus and damping, yielding at failure and no-tension cut-off. The response of the pile groups is computed

directly without having to use the p-y curves and the p-multipliers.

#### 2. FULL SCALE FIELD TESTS BY HUANG ET AL.

As shown in Figure 1, the bored pile group consisted of 6 piles and the PC pile group consisted of 12 piles. Piles in each group were cast into a solid pile cap as detailed in Huang et al. (2001). A jack and load cell system was installed between the two pile groups to apply the lateral forces for lateral load tests on the two pile groups. Lateral load tests on single piles were conducted on piles B7, B13 and P7. All piles were spaced at a centre-to-centre distance of 3 times the diameter of piles in each group. The PC piles are hollow but infilled with concrete. The structural properties of both the bored piles and the PC piles are listed in Table 1.

Table 1 S <sup>.</sup>	tructural Prope	rties of Piles	(Huang et al.	. 2001)

Item	Bored piles	PC piles
Diameter	1.5 m	0.8 m OD, 0.56 m ID
		With concrete infill
Length	34.9 m	34 m
Intact flexural rigidity, El	6.86 x 10 <sup>6</sup> kN.m <sup>2</sup>	0.79 x 10 <sup>6</sup> kN.m <sup>2</sup>

The test site was located within a sugarcane field. Soil investigation before and after the pile installations consisted of 8 boreholes, 6 cone penetration tests (CPT), and 5 dilatometer tests (DMT). Approximate locations of

the CPT holes are shown in Figure 1 and results of the CPT tests are shown in Figure 2. According to soil samples recovered from the boreholes, the soils within the 80 m depth were generally classified as silty sand to

silt with occasional layers of silty clay. The groundwater table was located at approximately 1 m below the ground surface.



Figure 1. Arrangement of Test Piles and CPT Test Holes, after Huang et al. (2001)



Figure 2. Cone Penetration Test - CPT Data from the Site, after Huang et al. (2001)

- 3. VERIFICATION OF THE QUASI-3D FINITE ELEMENT METHOD
- 3.1 The Quasi-3D Finite Element Method

Under vertically propagating shear waves (Figure 3) the foundation soils undergo mainly shearing deformations in the XoY plane, except in the area near the pile where extensive compression deformations develop in the direction of shaking. The compressive deformations also generate shearing deformations in the YoZ plane as shown in Figure 3.

The quasi-3D method proposed by Wu (1994) and Wu and Finn (1997a, 1997b) assumed that dynamic motions are governed by shear waves in the XoY and YoZ planes and compression waves in the direction of shaking, Y. Thus they developed the following equation for describing the free vibration of the soil continuum:

[1] 
$$\rho_s \frac{\partial^2 v}{\partial t^2} = G^* \frac{\partial^2 v}{\partial x^2} + \theta G^* \frac{\partial^2 v}{\partial y^2} + G^* \frac{\partial^2 v}{\partial z^2}$$

Where v is displacement in the Y direction,  $\rho_s$  is the mass density of soil, and G is a complex shear modulus. The complex shear modulus G is expressed as G =G (1+i·2 $\lambda$ ) in which G is the shear modulus of soil, and  $\lambda$  is the hysteretic damping ratio of the soil. The parameter  $\theta$  was derived to be  $\theta$ =2/(1- $\mu$ ) assuming a plane strain condition in the Y direction (Wu and Finn, 1999), and  $\mu$  is Poisson's ratio of the soil.





The quasi-3D method is further enhanced in VERSAT-P3D (Wu, 2006) by implementing 8-node pile element to account for pile diameter effect and energy transmitting boundaries for analysis of high frequency vibrations such as machine foundation response. An 8-node pile element is a bending structural element that consists of four beams arranged at the four sides of an 8-node brick that has the size of a pile.

The VERSAT-P3D finite element code has improved the quasi-3D method in two aspects. The results of calculation are less dependent on the size of the finite element grids, in relative to the pile diameter, and they are stable and reliable for analysis of high frequency vibrations.

#### 3.2 Verification of the Quasi-3D Method

For verification purposes, dynamic stiffnesses at the pile head for single piles in an assumed elastic soil medium having a shear wave velocity of V<sub>s</sub> were computed as a function of dynamic load frequency,  $\omega$ , using VERSAT-P3D. The analyses were conducted for a pile-soil system with E<sub>p</sub>/E<sub>s</sub> of 1000, Poisson's ratio  $\mu$  of 0.4 and hysteretic damping ratio  $\lambda$  of 5% for the soil, where E<sub>p</sub> and E<sub>s</sub> are the Young's modulus of the pile and soil, respectively. The computed horizontal stiffnesses, normalized to E<sub>s</sub>d, are compared in Figure 4 with those reported in Wu and Finn (1997a) and Kaynia and Kausel (1982). As compared to d=0.3 m in Figure 4, further analyses have shown that the results are repeatable for a larger diameter pile with d=0.76 m.

The analyses showed that the standard grid of 8x24, a built-in module in VERSAT-P3D for analyses of single piles, produces results that are very satisfactory until the dimensionless frequency  $a_0$  (= $\omega d/V_s$ ) reaches about 0.6, or a vibration frequency of 180 rad/sec for a site with  $V_s=90$  m/s and a pile diameter of 0.3 m. A pile-soil system involving high frequency vibrations (a0 > 0.6) may require the use of a finite element grid for high frequency, which employs equal size for each grid outside the piles. Grid size of a standard grid increases with the distance of grid line from the piles and is the largest at the model boundaries. As a general role of thumb, a standard grid is good for large strain nonlinear analysis with relatively low frequency, such as for seismic loading conditions, where the application requires a far-field (or free field) zone to differentiate it from the influence of the piles. But the standard grid is not satisfactory for analysis of highfrequency vibrations as the large grid size can often truncate the high frequency response.

Dynamic stiffnesses of a 2x2 pile group with a spaceto-diameter ratio of s/d=5 were computed and compared with those by Wu and Finn (1997a) and Kaynia and Kausel (1982). For some yet unknown reasons, the peak horizontal stiffnesses from this study occur at a higher  $a_0$  of 0.9 instead of 0.8 as obtained by Kaynia and Kausel (1982). Otherwise, the results are in good agreement.

# 4. ANALYSIS OF FULL SCALE FIELD TESTS BY HUANG ET AL.

#### 4.1 Soil Parameters

The soil profile and parameters, derived from the borehole, CPT and DMT data of the site, are shown in Table 2, which were used for all analyses reported herein. In addition, curves for shear modulus reduction and hysteretic damping versus shear strain (Seed et al., 1986) were used in the nonlinear analyses of the pile response subjected to high magnitude of loads such as the tests of Huang et al. (2001).

4.2 Analyses for Single PC and Bored Piles

A standard grid of 8x24 was used for the analysis of single piles. The 3D model sizes in plan were 42.9 m x 134 m for the bored pile (B7) and 22.4 x 70 m for the PC pile (P7). The depth of the model is 39 m for both.

As noted by Huang et al. (2001), the field inclinometer measurements showed a sharp change in curvature in P7 at a lateral load of 570 kN and in B7 at a lateral load of 1462 kN, indicating most likely the piles cracked at these loads. Reduced flexural rigidity (EI) of piles was assigned to part of the piles to account for the influence of section cracking, as shown in Table 3. It is noted that response at a load level of 422 kN (P7) and 1152 kN (B7) was not analyzed because the author was not certain whether the piles had cracked or not. The reduced values of EI used in this study (Table 3) are the same as in Huang et al. for P7. The reduced EI is  $1.37x10^{6}$ kN.m<sup>2</sup> for B7, which is smaller than 4.29 to  $5.75 x10^{6}$  kN.m<sup>2</sup> of Huang et al.

The results of the single pile analyses are shown in Figure 6 for pile head deflection versus load level, and in Figure 7 for pile deflection profile versus depth. They are in good agreement with the measured pile deflections from the tests.

#### 4.3 Analyses of the Pile Groups

Standard grids were used for the analyses of pile groups. The 3D model sizes in plan were  $24.8 \times 77.2$  m for the PC pile group with twelve (3x4) piles and a pile cap of approximately 6 m x 8.4 m, as shown in Figure 8, and 45.2 x 143.9 m for the bored pile group with six (2x3) piles and a pile cap of 8 m x 12 m. The depth of the model is 39 m for both.



Figure 4 comparison of normalized horizontal stiffness for single piles ( $E_p/E_s$ =1000,  $\mu$ =0.4 and  $\lambda$ =5%)



Figure 5 Group factors for horizontal stiffness for a 2 x 2 pile group ( $E_p/E_s=1000$ ,  $\mu=0.4$  and  $\lambda=5\%$ )

soil	depth <sup>1</sup>	soil	soil density	shear modulus	Poisson's	tension strength	yield strength
layer no.	(m)	type	(kN/m³)	(kPa)	ratio, μ	(kPa)	(kPa)
M3	1	sandy silt	18	5700	0.4	11	10
M4	2	sandy silt	18	5700	0.4	15	14
M5	3	sandy silt	18	5700	0.4	19	18
M6	4.2	silty sand	19.5	33300	0.35	48	34
M7	5.4	silty sand	19.5	33300	0.35	65	45
M8	6.5	silty sand	19.5	33300	0.35	65	45
M9	8	silt	18	10300	0.45	VH <sup>2</sup>	VH <sup>2</sup>
M10	9.5	silt	18	10300	0.45	VH <sup>2</sup>	VH <sup>2</sup>
M11	11	silt	18	10300	0.45	VH <sup>2</sup>	VH <sup>2</sup>
M12	12.5	silty sand	19.5	48000	0.35	VH <sup>2</sup>	VH <sup>2</sup>
M13	14.5	silty sand	19.5	48000	0.35	VH <sup>2</sup>	VH <sup>2</sup>
M14	18	silt&sand	19	48000	0.4	VH <sup>2</sup>	VH <sup>2</sup>
M14	22	silt&sand	19	48000	0.4	VH <sup>2</sup>	VH <sup>2</sup>
M15	26	silt	18.5	11000	0.45	VH <sup>2</sup>	VH <sup>2</sup>
M15	30	silt	18.5	11000	0.45	VH <sup>2</sup>	VH <sup>2</sup>
M16	33	silty sand	20	48000	0.35	VH <sup>2</sup>	VH <sup>2</sup>
M16	39	silty sand	20	48000	0.35	VH <sup>2</sup>	VH <sup>2</sup>

Table 2 Soil profile and parameters used in VERSAT-P3D analyses

<sup>1</sup> depth is from the underside of the pile cap

<sup>2</sup> very high (VH) values are assigned to the soil layers when they are considered to be non-yield or non-tension materials

load level, kN	E I of P7 (kN.m <sup>2</sup> )		load level kN	E I of B7 (kN.m <sup>2</sup> )	
	depth 4 to 6 m	other depth		depth 6 to 9 m	other depth
265	0.79 x 10 <sup>6</sup>	0.79 x 10 <sup>6</sup>	814	6.86 x 10 <sup>6</sup>	6.86 x 10 <sup>6</sup>
570	$0.14 \ge 10^{6}$	0.79 x 10 <sup>6</sup>	1462	$1.37 \ge 10^6$	6.86 x 10 <sup>6</sup>
736	$0.10 \ge 10^{6}$	0.79 x 10 <sup>6</sup>	1908	$1.37 \ge 10^{6}$	6.86 x 10 <sup>6</sup>
			2943	1.37 x 10 <sup>6</sup>	6.86 x 10°

Table 3 Flexural rigidity of single piles used in the analysis



Figure 6 Comparison of lateral load versus pile head deflection for single piles



Figure 7 Comparison of pile deflection versus depth for single piles

As noted in Huang et al. (2001), there was no apparent sign of pile breakage during the load tests (thus 100% El), but the pile-cap connection was not always rigid throughout the tests due to excessive large moment induced at the pile head that exceeds the moment capacity of the pile. Inclinometer readings showed relatively large rotation at the PC pile-cap connection from the beginning of the test, but at the bored pile-cap connection at a load level of 9643 kN. Thus the rocking stiffness of the pile cap due to vertical resistance of piles was reduced in the analysis of the pile group response (note: it becomes zero for a pinned pile group), as shown in Table 4.

The results of pile group analyses are shown in Figures 8 and 9. In general they are in good agreement with the measured pile deflections from the tests. It is noted that the computed pile head deflections of the PC pile group are slightly higher at 2774 kN and 5550 kN, but slightly lower at 7122 kN, than the measured values. The assumed  $K_{rock}$ -transfer value is considered to be an average of the three. The results may be better matched by using a slightly higher value at 2774 kN, but lower value at 7122 kN, of the transferred rocking stiffness. The average value was selected to show that the results of analysis are not very sensitive to  $K_{rock}$ -transfer used.

#### 4.4 Summary

The quasi-3D finite element method for response analysis of single piles or pile group is a relatively straight forward method. The results presented in the paper demonstrate that the method can model the single pile response in an accurate manner and can capture the pile group behaviour, under linear elastic or nonlinear large strain, without using p-y curves or p-multipliers to account for pile-soil interactions.

#### 5. DISCUSSIONS

#### 5.1 P-y curves and P-multipliers

Huang et al. (2001) used p-y curves and p-multipliers in their analyses of the load tests. Their key findings are summarized in Table 5. In order to match the loaddeflection data from the lateral load tests on single piles, the theoretically derived p-y curves using the field DMT data were required to be factored by 0.21 and 0.5 for P7 (PC pile) and B7 (bored pile), respectively. In other words, the soil lateral resistance to the pile has to be reduced by almost 5 times for P7, which is indeed a very significant correction on the p-y curves for a single pile.

When theoretical p-y curves are used, the p-multipliers for the PC pile group range from 0.23 to 0.33 in order to match the results of load tests, which is an awful low number to be applied in routine engineering work without a load test. In contrast, the p-multiplier could be as high as 1.08 to 1.57 when the actual p-y curves calibrated by the load tests are used. They appeared to have difficulties explaining the unusual result of  $p_m$  being consistently > 1.0.

Mostafa and El Naggar (2002) computed dynamic pmultipliers of the two pile groups reported in Huang et al. (2001). They obtained an average p-multiplier of 0.94 and 0.75 for the PC pile group and the bored pile group, respectively. From the paper, it is not clear how the static p-y curves were derived, either based on DMT data or based on field load tests. It is noticed that, in the paper, the classic p-multiplier instead of the back-calculated pmultiplier by Huang et al. (2001) were cited and compared with their own results.

5.2 Quasi-3D Finite Element Approach

Compared to the traditional approach using the p-y curve and p-multiplier, the quasi-3D finite element method is considered to be a more fundamental and straight forward approach in solving problems of pile-soil interactions. This approach uses only the fundamental parameters of the pile-soil system such as the size and stiffness of the pile, and stiffness and strength of the surrounding soils.

In contrast, the p-y curve approach, and the p-multiplier approach for a pile group, contains a variety of factors (such as variation of p-y curve and p-multiplier with pile diameter and soil nonlinearity) that are simply too complex to be determined reliably and accurately in practice.



Figure 8 Quasi-3D finite element model used for the analysis of a 3x4 PC pile group

Table 4 Flexural rigidity of pile and transfer factor for rocking stiffness of pile cap

load level, kN	PC pile group		load lovel kN	Bored pile group	
	K <sub>rock</sub> -transfer	E I for piles		K <sub>rock</sub> -transfer	E I for piles
2774	0.025	0.79 x 10 <sup>6</sup>	3284	1.00	6.86 x 10 <sup>6</sup>
5550	0.025	0.79 x 10 <sup>6</sup>	6568	1.00	6.86 x 10 <sup>6</sup>
7122	0.025	0.79 x 10 <sup>6</sup>	8348	1.00	6.86 x 10 <sup>6</sup>
Unit for EI: kN.m <sup>2</sup>			10948	0.35	6.86 x 10 <sup>6</sup>

Note: Rocking stiffness of a pile group = rotational stiffness of piles + K<sub>rock</sub>-transfer \* rocking stiffness of pile cap



Figure 9 Comparison of lateral load versus pile head deflection for pile groups



Figure 10 Comparison of pile deflection profile versus depth for pile groups

	PC pile group	- 3 x 4; p <sub>mga</sub> =0.37 (p	pre-installation)	Bored pile group - 2 x 3; p <sub>mga</sub> =0.47 (pre-installation)		
pile row	f <sub>m</sub>	p <sub>m0</sub> <sup>(1)</sup>	p <sub>m</sub> <sup>(2)</sup>	f <sub>m</sub>	p <sub>m0</sub> <sup>(1)</sup>	p <sub>m</sub> <sup>(2)</sup>
	classic p <sub>multiplier</sub>	$p_{m0} = f_m * p_{mga}$	$p_{m1} = f_m * p_{mga} / p_{ms}$	classic p <sub>multiplier</sub>	$p_{m0}=f_m^*p_{mga}$	$p_{m1} = f_m * p_{mga} / p_{ms}$
leading	0.893	0.33	1.57	0.932	0.44	0.88
middle	0.614	0.23	1.08	0.704	0.33	0.66
middle	0.614	0.23	1.08			
trailing	0.660	0.24	1.16	0.740	0.35	0.70

Table 5 P-multipliers back-calculated from lateral load tests on single piles and pile groups (Huang et al., 2001)

 $p_{ms}$  -  $p_{multiplier}$  used to correct p-y curves (DMT data) to p-y curves (load test);  $p_{ms}$  =0.21 and 0.50 for PC and bored piles, respectively  $p_{m0}^{(1)}$  -  $p_{multiplier}$  to be applied on p-y curves theoretically derived using DMT data

 $p_{m0}$  -  $p_{multiplier}$  to be applied on p-y curves theoretically derived using Divit data

 $p_m{}^{(2)}\,$  -  $\,p_{multiplier}\,$  to be applied on p-y curves calibrated on single pile load tests

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